

Apportionment Problems

Lecture 19 Section 4.1

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- 1 Apportioning Candies
- 2 The House of Representatives
- 3 Hamilton's Solution
- 4 Assignment

Outline

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- 2 The House of Representatives
- 3 Hamilton's Solution
- 4 Assignment

Apportioning Candies

Example

- I have a class of 5 students and I have 50 pieces of candy to hand out.

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- Ok, I ate them. Still, they're gone.
- Now there are 400 total points and 47 pieces of candy.
- So, how many candies should each student get?

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- How many seats should each state get?
- $323,000,000 \div 435 = 742,529$.
- Each seat should represent 742,529 people.

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- So, Virginia gets 11.329 seats.
- Obviously, that is not possible, either.
- What to do?

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 - Calculate the **exact** number of seats that each state deserves, based on its population.
 - Separate each of those numbers into a **whole number** and the **fractional part**.
 - Give each state its whole number of seats.
 - Distribute the remaining seats to those states with the largest fractional parts.

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- Give the students the whole numbers of candies

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for a total of 45 candies.

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- Which students should get the remaining 2 candies?

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- Congress failed to override Washington's veto.
- What happened next?

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- Chapter 4: Exercises 11, 12, 13, 14. Skip the terminology; apply Hamilton's method.